

*This essay received an Honorable Mention in the Gravity Research Foundation Essay Competition 2015.*

## **Ten shades of black**

Shahar Hod

*The Ruppin Academic Center, Emeq Hefer 40250, Israel*

*and*

*The Hadassah Institute, Jerusalem 91010, Israel*

(Dated: December 4, 2015)

### **Abstract**

The holographic principle has taught us that, as far as their entropy content is concerned, black holes in  $(3 + 1)$ -dimensional curved spacetimes behave as ordinary thermodynamic systems in flat  $(2 + 1)$ -dimensional spacetimes. In this essay we point out that the *opposite* behavior can also be observed in black-hole physics. To show this we study the quantum Hawking evaporation of near-extremal Reissner-Nordström black holes. We first point out that the black-hole radiation spectrum departs from the familiar radiation spectrum of genuine  $(3 + 1)$ -dimensional perfect black-body emitters. In particular, the would be black-body thermal spectrum is distorted by the curvature potential which surrounds the black hole and effectively blocks the emission of low-energy quanta. Taking into account the energy-dependent gray-body factors which quantify the imprint of passage of the emitted radiation quanta through the black-hole curvature potential, we reveal that the  $(3 + 1)$ -dimensional black holes effectively behave as perfect black-body emitters in a flat  $(9 + 1)$ -dimensional spacetime.

Email: shaharhod@gmail.com

**Introduction.** — The holographic principle has revealed that, as far as their thermodynamic properties are concerned, black holes in  $(3 + 1)$ -dimensional spacetimes are fundamentally  $(2 + 1)$ -dimensional objects [1–3]. In particular, the entropy content of a black hole in 3-D space scales with the black-hole 2-D surface area [4]. In this essay we reveal that the *opposite* behavior can also be observed in the physics of black holes.

To that end, we study the quantum Hawking evaporation of near-extremal Reissner-Nordström (RN) black holes. We shall show that the effective curvature potential which surrounds the evaporating black holes distorts the emitted Hawking quanta in such a way that the resulting black-hole radiation spectrum is no longer that of a perfect 3-D black-body emitter. Moreover, a detailed analysis (to be carried out below) reveals that these  $(3 + 1)$ -dimensional black holes effectively behave as perfect black-body emitters in a flat  $(9 + 1)$ -dimensional spacetime.

**Quantum evaporation of near-extremal RN black holes.** — Hawking’s celebrated result that black holes emit a thermally distributed radiation is certainly one of the most important theoretical predictions of modern physics [5]. Although the Hawking black-hole radiation spectrum has distinct thermal features, it is important to realize that it departs from the familiar radiation spectra of perfect black-body emitters. In particular, the would be black-body thermal spectrum is distorted by the curvature potential which surrounds the black hole and effectively blocks the emission of low-energy quanta. The departure of the Hawking black-hole radiation spectrum from the pure spectrum of a perfect (thermal) black-body emitter can be quantified by the frequency-dependent gray-body factors  $\{\Gamma(\omega)\}$  [6].

In this essay we shall explore the power spectrum of the coupled electromagnetic-gravitational quanta emitted by near-extremal Reissner-Nordström (RN) black holes. The Hawking temperature of these  $(3 + 1)$ -dimensional black holes is given by [7]

$$T_{\text{RN}} = \frac{\hbar \sqrt{M^2 - Q^2}}{2\pi(M + \sqrt{M^2 - Q^2})^2} , \quad (1)$$

where  $M$  and  $Q$  are the black-hole mass and electric charge, respectively. The Hawking radiation power out of the black holes is given by [8]

$$P_{\text{RN}}^{3+1} = \frac{T_{\text{RN}}}{2\pi} \sum_{l,m} \int_0^\infty d\omega \frac{\Gamma x}{e^x - 1} , \quad (2)$$

where  $x \equiv \hbar\omega/T_{\text{RN}}$ . Here  $l$  and  $-l \leq m \leq l$  are the harmonic indexes of the emitted

quanta, and  $\Gamma = \Gamma_{lm}(\omega)$  are the energy-dependent gray-body factors [6]. These dimensionless transmission coefficients quantify the imprint of passage of the emitted radiation through the effective curvature potential which surrounds the black hole.

The thermal factor that appears in the denominator of (2) implies that the black-hole emission spectrum peaks at the characteristic frequency  $x^{\text{peak}} \equiv \hbar\omega^{\text{peak}}/T_{\text{RN}} = O(1)$ . Remembering that near-extremal black holes are characterized by the relation  $MT_{\text{RN}}/\hbar \ll 1$ , one finds the strong inequality

$$M\omega^{\text{peak}} \ll 1 \quad (3)$$

for the characteristic frequencies emitted by the near-extremal black holes.

The relation (3) implies that, for near-extremal black holes, the typical wavelengths in the Hawking radiation spectrum are very large on the scale set by the geometric size of the evaporating black hole. The calculation of the frequency-dependent grey-body factors  $\Gamma_{lm}(\omega)$  in the low-frequency regime (3) is a common practice in the physics of black holes [6]. In particular, one finds the leading-order behavior [9]

$$\Gamma_{11} = \Gamma_{2m} = \frac{4}{9}(\omega r_{\text{H}})^8 \quad (4)$$

in the small-frequency regime  $\omega r_{\text{H}} \ll 1$ , where  $r_{\text{H}} \simeq M$  is the outer horizon radius of the near-extremal black hole [10]. Substituting the frequency-dependent grey-body factors (4) into (2) and performing the integration, one finds

$$P_{\text{RN}}^{3+1} = C_{\text{RN}}^{3+1} \times \frac{r_{\text{H}}^8 T_{\text{RN}}^{10}}{\hbar^9} \quad (5)$$

for the emission power out of the RN black holes [11].

Evidently, the Hawking radiation power (5) out of the  $(3+1)$ -dimensional near-extremal black holes looks completely different from the familiar Stefan-Boltzmann law [12]

$$P_{\text{flat}}^{3+1} = C_{\text{flat}}^{3+1} \times \frac{R^2 T^4}{\hbar^3} \quad (6)$$

for perfect black-body emitters of temperature  $T$  and radius  $R$  in a  $(3+1)$ -dimensional flat spacetime [13].

We shall now prove, however, that the Hawking radiation power (5) characterizing the  $(3+1)$ -dimensional near-extremal black holes is of the *same* functional form as the thermal radiation power of perfect black-body emitters in a *higher*-dimensional flat spacetime.

**Perfect black-body emitters in flat  $(D+1)$ -dimensional spacetimes.** — We shall now obtain the thermal radiation power which characterizes perfect black-body emitters in general  $(D+1)$ -dimensional flat spacetimes.

To that end, we first note that the thermal energy density of one bosonic degree of freedom inside a  $(D+1)$ -dimensional closed cavity of temperature  $T$  is given by [14, 15]

$$\rho_D = \frac{T}{(2\pi)^D} \int_0^\infty dV_D(\omega) \frac{x}{e^x - 1}, \quad (7)$$

where  $x \equiv \hbar\omega/T$  and  $dV_D(\omega) = [2\pi^{D/2}/\Gamma(D/2)]\omega^{D-1}d\omega$  is the volume in the  $(D+1)$ -dimensional frequency-space of the shell  $(\omega, \omega + d\omega)$ . Substituting  $dV_D(\omega)$  into (7) and performing the integration, one finds the  $(D+1)$ -dimensional thermal energy density

$$\rho_D = \frac{\Gamma(D+1)\zeta(D+1)}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \times \frac{T^{D+1}}{\hbar^D}, \quad (8)$$

where  $\zeta(z)$  is the Riemann zeta function [16]. Since the thermal radiation is emitted from a sphere of surface-area  $A_{D-1} = [2\pi^{D/2}/\Gamma(D/2)]R^{D-1}$  [17], the radiated power  $P_{\text{flat}}^{D+1}$  out of the  $(D+1)$ -dimensional perfect black-body emitter is proportional to  $\rho_D \times A_{D-1}$ , which yields [15, 18]:

$$P_{\text{flat}}^{D+1} = C_{\text{flat}}^{D+1} \times \frac{R^{D-1}T^{D+1}}{\hbar^D}. \quad (9)$$

A direct comparison between the two radiation powers, Eqs. (5) and (9), reveals the surprising conclusion that our  $(3+1)$ -dimensional near-extremal black holes effectively behave as perfect black-body emitters in a flat  $(9+1)$ -dimensional spacetime.

**Summary.** — In this essay we have analyzed the Hawking emission of coupled electromagnetic-gravitational quanta by near-extremal Reissner-Nordström black holes. It was pointed out that, due to the influence of the energy-dependent gray-body factors which quantify the imprint of passage of the emitted quanta through the spacetime curvature potential, the black-hole radiation spectrum departs from the familiar radiation spectrum of genuine  $(3+1)$ -dimensional perfect black-body emitters. In particular, it was shown that the curvature potential which surrounds these  $(3+1)$ -dimensional near-extremal black holes distorts the emitted Hawking spectrum in such a way that the resulting black-hole radiation power is effectively that of a perfect black-body emitter in a flat  $(9+1)$ -dimensional spacetime.

*Acknowledgments:* This research is supported by the Carmel Science Foundation. I thank Yael Oren, Arbel M. Ongo, Ayelet B. Lata, and Alona B. Tea for stimulating discussions.

- 
- [1] G. 't Hooft, in *Salam-festschrift*, ed. A. Aly, J. Ellis and S. Randjbar-Daemi (World Scientific, Singapore 1993) [arXiv:grqc/9310026].
  - [2] L. Susskind, J. Math. Phys. **36**, 6377 (1995).
  - [3] J. D. Bekenstein and A. E. Mayo, Gen. Rel. Grav. **33**, 2095 (2001) [This essay received Second Award in the Annual Essay Competition of the Gravity Research Foundation for the year 2001].
  - [4] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
  - [5] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
  - [6] D. N. Page, Phys. Rev. D **13**, 198 (1976).
  - [7] We use gravitational units in which  $G = c = k_B = 1$ .
  - [8] W. H. Zurek, Phys. Rev. Lett. **49**, 1683 (1982); D. Page, Phys. Rev. Lett. **50**, 1013 (1983).
  - [9] L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Phys. Rev. D **82**, 124038 (2010).
  - [10] Note that the emission of quanta with larger values of the spherical harmonic index  $l$  is suppressed due to the fact that the corresponding grey-body factors of these higher- $l$  modes contain higher powers of the small quantity  $\omega r_H \ll 1$  [9].
  - [11] The dimensionless coefficient in (5) is given by  $C_{\text{RN}}^{3+1} = 1024\pi^9/297$ .
  - [12] C. W. Allen, *Astrophysical Quantities* (Athlone Press, London, 1973), 3rd ed.
  - [13] The dimensionless coefficient in (6) is given by  $C_{\text{flat}}^{3+1} = \pi^3/30$  [12].
  - [14] J. D. Bekenstein, Phys. Rev. D **49**, 1912 (1994); S. Hod, Phys. Lett. B **695**, 294 (2011); S. Hod, Phys. Lett. B **700**, 75 (2011).
  - [15] T. R. Cardoso and A. S. de Castro, Rev. Bras. Ens. Fis. **27**, 559 (2005).
  - [16] Here we have used the relation  $\int_0^\infty dx x^D/(e^x - 1) = \Gamma(D+1)\zeta(D+1)$ .
  - [17] Note that the radiating surface-area is of  $D-1$  spatial dimensions.
  - [18] The dimensionless coefficient in (9) is given by  $C_{\text{flat}}^{D+1} = D\zeta(D+1)/\pi$ . Here we have used the identity  $\Gamma(2z) = \pi^{-1/2}2^{2z-1}\Gamma(z)\Gamma(z+1/2)$ .